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COMPUTER GRAPHICS METHOD FOR SOLVING TRANSCENDENTAL EQUATIONS

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A COMPUTER GRAPHICS METHOD

FOR SOLVING TRANSCENDENTAL EQUATIONS

by

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TABLE OF CONTENTS

LIST OF FIGURES

Introduction

Finding the roots of an equation $F(x)=0$ when $F(x)$ involves transcendental functions and x is complex usually involves some kind of search method. The efficiency of a search method depends to a certain extent on knowledge of the roots--where they are likely to occur in the x plane, and how many there are. If a root is known to lie in a given region in the x plane, then a search routine can quickly find the root to the desired accuracy. But if no information about the location of the roots is available, a search over a wide area must be conducted, and this can be time consuming and expensive. Consequently, a method for locating the general area of the roots and determining the pattern of the roots is very valuable.

This report describes a simple method for graphically displaying the pattern of roots in the complex plane. The advantages of this method are the same as the advantages of computer graphics in general. It provides a way of visualizing the system, in this case, the transcendental equation; and it allows one to obtain a qualitative feel for the behavior of the system and therefore intuition about the system.

Determining the Root of a Transcendental Equation

There are a number of ways to determine the values of x which satisfy the equation $F(x)=0$. (The case of interest here is that in which F is a complex function of the complex variable x.) One way is to search for values of x which make Real $(F(x))$ and Imaginary $(F(x))$ both zero at the same time where the search is based on sign changes in both quantities. There is a problem in that a sign change occurs across a pole as well as a zero.

The method used here is that the transcendental equation is written in the form

$$f(x) = 1 \tag{1}$$

and f(x) is written in the form

$$f(x) = R(x)e^{i\phi(x)} \tag{2}$$

where R(x) is the magnitude of f(x) and $\phi(x)$ is the phase. The roots of Equation 1 are values of x which make

$$R(x) = 1$$
$$\phi(x) = 2m\Pi \quad m = 0, \pm 1, \pm 2, \ldots$$

Let

$$A(x) = 0.5 \text{ if } R(x) \geq 1$$
$$A(x) = 0 \quad \text{if } R(x) \leq 1$$
$$P(x) = \frac{0.5\phi}{2\Pi} \quad 0 \leq \phi < 2\Pi$$
$$B(x) = A(x) + P(x)$$

Now B(x) is to be the brightness on a graphics display. Since A(x) and P(x) each have discontinuities, B(x) will show two kinds of discontinuities. The solutions to Equation 1 occur where the discontinuities of A(x) and P(x) occur simultaneously. These points may be seen on a display of B(x). An example is shown in Figure 1

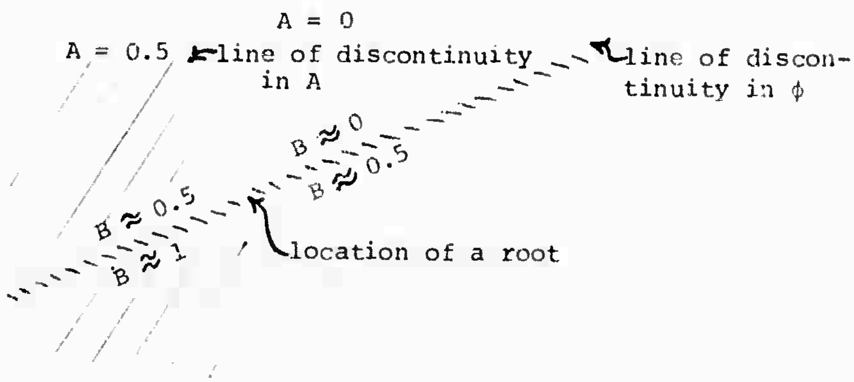

Figure 1: Illustration of a display of B(x).

A computer program can be written to compute and display B(x) for a grid of points in the x plane. The roots of Equation 1 are then immediately obvious, but more important, the pattern of the roots in the complex plane can be seen, and a qualitative feel for the transcendental equation can be obtained.

A Graphics Display

The method described above was used to obtain the computer-generated photograph shown in Figure 2. The transcendental equation is

$$\frac{e^{-y}}{x-(1+i0.02)} = 1$$

A computer program (see printout in Appendix A) was written to calculate and display B(x) for 131x131 points in the complex x plane in increments of 0.1. Coordinate lines spaced 3 units in x are also shown. The location of three of the roots is readily apparent. One of them, which is not quite obvious at first glance, is almost on the real axis near 1.3. An infinite number of complex roots occur upwards and downwards on the line of discontinuity in A, which is the line in the general up and down direction. The lines which go across the photograph are discontinuities in P. This photograph shows how the pattern of the roots is readily identifiable. A search routine could be used to find two roots as accurately as desired, and very efficiently, because their approximate location can be easily obtained from the photograph.

The photograph shown in Figure 2 was generated by a computer graphics system of the Computer Science Division at the University of Utah utilizing a multiple point-by-point exposure. A diagram of the system is shown in Figure 3. The approximate Univac 1108 computer time used to make the photograph was 170 seconds.

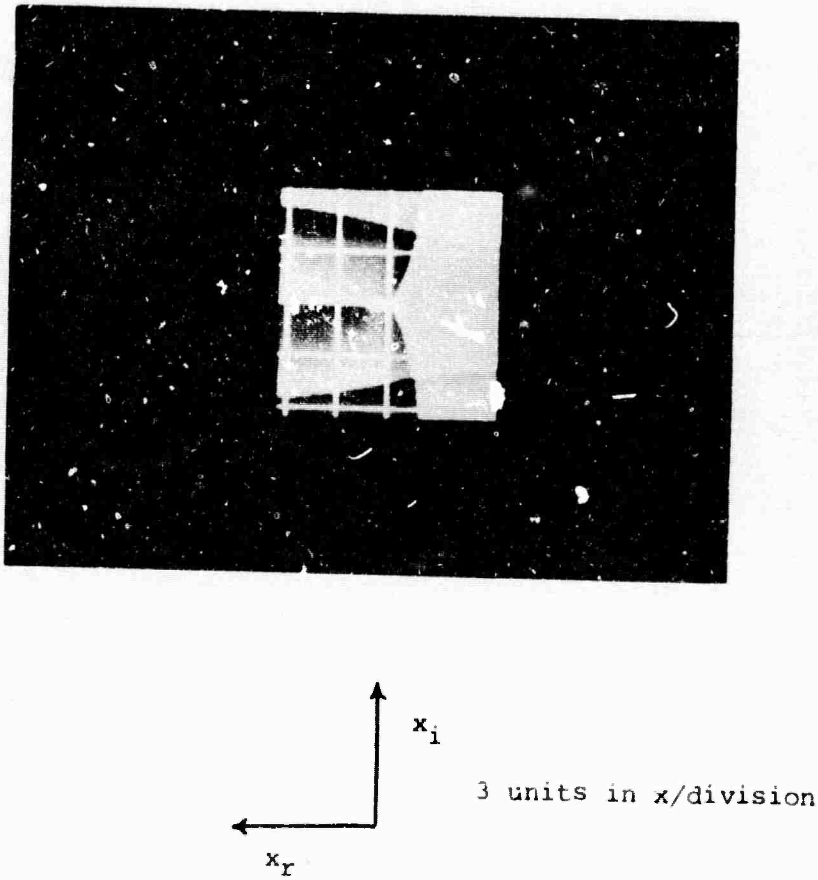

Figure 2: A computer-generated photograph showing B(x) in the complex x plane. There are 131x131 points corresponding to increments in x of 0.1. The roots occur at the intersections of the lines of discontinuity. Three roots are shown in this photograph. One of them is almost on the real axis.

A Poor Man's Graphic Method

All the information contained in Figure 2 is also contained in the printout shown in Figure 3, even though the printout is not as pretty. The direction of real x is reversed from that of the photograph. The same basic method was used to obtain the printout in Figure 3, but the program is much simpler because no graphics system is involved, and the method can be used with any digital computer system. The program listing is given in Appendix B.

In the printout shown in Figure 3 each space represents a complex value of x and the increments used here are 0.1. A slash in a space indicates $R(x) \geq 1$. The digitized value of $\phi(x)$ are indicated as:

Blank	0
Period	1
2	2
Comma	3
4	4
Dash	5
6	6

Since the value of ϕ was calculated in floating point and then converted to fixed point, a 2 means $2.0 \leq \phi < 3.0$. The locus of points where ϕ changes from 2Π to 0 are readily apparent, and the other symbols show how ϕ is changing over the plane. Since the program is extremely simple, this method should be a very useful tool, even to those without access to a graphics system. The 1108 computer time used to obtain the printout of Figure 3 was 13 seconds. The method

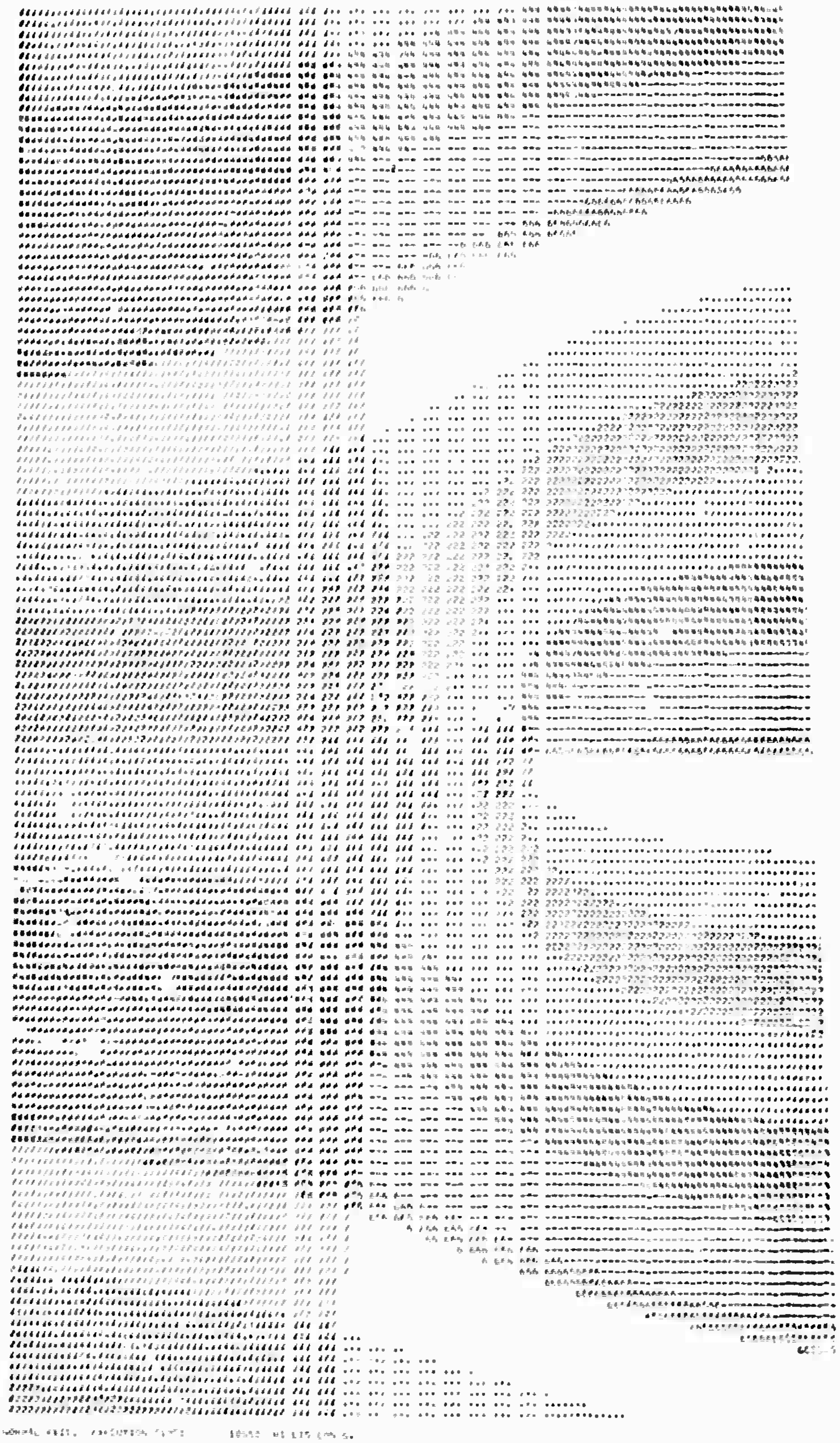

Figure 3: A printout showing the same information as the photograph in Figure 2.

was used here at the University of Utah with good success in solving the dispersion equation describing the double-stream microwave interaction between electrons and holes in semiconductors.[1] A little experimentation with density of characters would result in a more pleasing printout.

Conclusions

The inherent power of computer graphics has been applied to the solving of transcendental equations. The method described here is simple, yet powerful. It provides a way to visualize the characteristics of the transcendental equation and obtain insight into the nature of the roots. With this insight, a search routine could be used very efficiently to obtain the roots to the desired accuracy.

[1]Lewis C. Goodrich. A small-signal field analysis of double-stream interactions in finite semiconductors. Ph.D. thesis, University of Utah, Salt Lake City, Utah, June 1970.

APPENDIX A

This appendix contains the listing of the program used to obtain the photograph in Figure 2.

```
WELT,LIL BURNEY
ELT PROCESSOR LEVEL      3
      000      COMPLEX X,F
      000      INTEGER BF(171)
      000      NAMELIST/QUIT/EX
      000      CALL RELOAD
      000      CALL INOUT
      000      CALL SETEST
      000      READ(5,QUIT)
      000      CALL JUMPS('QUIT',$200)
      000      CALL CHRINT(1,58)
      000      CALL SWPCHR('X')
      000      CALL SWAP
      000      XR=-0.5
      000      XI=0.5
      000      K=6
      000      DO 100 I=1,512
      000      IF(I.GE.1.AND.I.LE.190)GO TO 100
      000      IB=0
      000      IW=1
      000      M=6
      000      DO 101 J=1,513
      000      IF(I.GE.322.AND.I.LE.512)GO TO 41
      000      IF(J.GE.1.AND.J.LE.190)GO TO 41
      000      IF(J.GE.322.AND.J.LE.513)GO TO 41
      000      IF((I-190).EQ.K.OR.(J-190).EQ.M)GO TO 40
      000      X=CMPLX(XR,XI)
      000      F=1./(CEXP(X)*(X-(1.0,.0200)))
      000      AM=CABS(F)
      000      R=REAL(F)
      000      AI=AIMAG(F)
      000      A=0
      000      IF(AM.GE.1.)A=50
      000      PHI=ATAN2(AI,R)
      000      IF(PHI.LT.0.)PHI=PHI+6.2831
      000      P=PHI*7.96
      000      BW=102.4*EXP(.023*(A+P))
```

```
000          XR=XR+.1
000          GO TO 5
000       40 BB=685.
000          M=M+30
000          XR=XR+.1
000          GO TO 5
000       41 BB=0
000        5 FLD(IB,12,F(IW))=FLD(24,12,INT(BB))
000          IF(IB.GE.24)GO TO 50
000          IB=IB+12
000          GO TO 101
000       50 IB=0
000          IW=IW+1
000      101 CONTINUE
000          IF((I-190).EQ.K)K=K+30
000          XR=-6.5
000          XI=XI-.1
000      100 CALL SNDTEX(DF)
000          CALL SWAP
000          CALL IDLE
000          GO TO 1
000      200 CALL EXIT
000        8 CALL TTY
000          CALL INIRET
000          END
```

FIN

APPENDIX B

This appendix contains the listing of the program used to obtain the printout given in Figure 3.

```
00101          COMPLEX X,F
00103          DIMENSION IP(131),IA(131 ,IC 7)
00104          DATA IC/1H , 1H., 1H2, 1 ,,  H4, 1H-  1H
00106          DATA M2/1H//, M3/1H
00111          CALL NOSKIP
00112          XR=-7.
00113          XI=7.
00114          DXI=-.1
00115          DXR=.1
00116          DO 6 J=1,131
00121          DO 1 I=1,131
00124          X=CMPLX(XR,XI)
00125          F=1./(CEXP(X)+(X-(1.,.62 0)))
00126          AM=CABS(F)
00127          IF(AM.GE.1.)IA(I)=M2
00131          IF(AM.LT.1.)IA(I)=M3
00133          R=REAL(F)
00134          AI=AIMAG(F)
00135          PHI=ATAN2(AI,R)
00136          IF(PHI.LT.0.)PHI=PHI + 6.283
00140          K=PHI
00141          IP(I)=IC(K+1)
00142        1 XR=XR + DXR
00144          PRINT 3,IP
00152        3 FORMAT(1X,131A1)
00153          PRINT 4,IA
00161        4 FORMAT(1H+,131A1)
00162          XR=-7.
00163        6 XI=XI+DXI
00165          STOP
00166          END
```